

Complete the worksheet on a separate sheet of paper. Solve each question below. The worksheet worth 100 points. Show all work.

1. Convert the parametric equations $x(t) = t^3(t-1)^3(t+1)^3$ and $y(t) = t^3 - t$ in a function of x and y .

2. Find the tangent line to the parametric curve $f(t) = (e^{\sqrt{2-t}}, t - \ln(t^2))$ at $t = -2$.

3. Find the tangent of the parametric equations $x(t) = \frac{e^t - t}{1+t^2}$ and $y(t) = \frac{t}{1+t^2}$ at $t = 2$.

4. Find all tangent lines to the parametric curve $f(t) = (t^2 - 2, t^3 - t)$ at the point $(-1, 0)$.

5. Find all points where the tangent line is horizontal or vertical given the parametric equations $x(t) = 2 \cos t$ and $y(t) = 4 \sin t$.

6. Find the net area of the parametric curve $f(t) = (\tan t, \sec t)$ on $0 \leq t \leq 1$.

7. Find the net area of the parametric equations $x(t) = t^3$ and $y(t) = (\ln t)^2 = \ln^2 t$ on $1 \leq t \leq \frac{e}{2}$.

8. Find the net area of the parametric equations $x(t) = (10 - t^2)^{100}$ and $y(t) = t^4$ on $0 \leq t \leq 3$.

9. Find the area enclosed by the parametric curve $f(t) = (\cos^3 t, -\sin^3 t)$ on $-\pi \leq t \leq \pi$. (Aside: For a better understanding consider the bounds $-4\pi \leq t \leq 4\pi$.)

10. Find the area between the parametric curve $f(t) = (\cos t, \sin^2 t)$ and the x -axis.

11. Find the area enclosed by the parametric equations $x(t) = 2 \cos(2t) + \cos(4t)$ and $y(t) = 2 \sin(2t) + \sin(4t)$. (Hint: Find a t such that $(x(\alpha), y(\alpha)) = (x(\beta), y(\beta))$, that is the initial point is equal to the terminal point.)

12. Find the arc-length of the parametric equations $x(t) = \frac{(2t+3)^{3/2}}{3}$ and $y(t) = t + \frac{t^2}{2}$ from $0 \leq t \leq 3$.

13. Find the arc-length of the parametric curve $f(t) = (re^{2t} \cos(2t), re^{2t} \sin(2t))$ from $0 \leq t \leq \pi$, where r is a positive Real Number constant $\left(r \in \{x \mid x \in \mathbb{R} \text{ and } x > 0\} \right)$.
14. Find the area A enclosed by the polar function $r = 3 + 2 \sin \theta$.
15. Find the area A enclosed by one loop of the polar function $r = \sin(4\theta)$. (*Hint: This problem is similar to the area enclosed by an inner loop problem, in this petal function each petal has equivalent area.*)
16. Find the area A enclosed by the loop of the polar function $r = 2 \cos \theta - \sec \theta$.
17. Find the area A enclosed by the polar function $r^2 = 9 \cos(2\theta)$. (*Aside: A function of this form is called a Lemniscate of Bernoulli.*)
18. Find the area A enclosed by any loop of the polar function $r = \theta \sin \theta$.
19. Find the arc-length L of the polar function $r = \theta^4$ from $0 \leq \theta \leq 2\pi$. (*Aside: A function of this form is called a Spiral of Archimedes.*)
20. Find the arc-length L of the polar function $r = 2 - 2 \cos \theta$.